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# Composing resilience techniques: ABFT, periodic and incremental checkpointing

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**Abstract**

Algorithm Based Fault Tolerant (ABFT) approaches promise unparalleled scalability and performance in failure-prone environments. Thanks to recent advances in the understanding of the involved mechanisms, a growing number of important algorithms (including all widely used factorizations) have been proven ABFT-capable. In the context of larger applications, these algorithms provide a temporal section of the execution, where the data is protected by its own intrinsic properties, and can therefore be algorithmically recomputed without the need of checkpoints. However, while typical scientific applications spend a significant fraction of their execution time in library calls that can be ABFT-protected, they interleave sections that are difficult or even impossible to protect with ABFT. As a consequence, the only practical fault-tolerance approach for these applications is checkpoint/restart. In this paper we propose a model to investigate the efficiency of a composite protocol, that alternates between ABFT and checkpoint/restart for the effective protection of an iterative application composed of ABFT-aware and ABFT-unaware sections. We also consider an incremental checkpointing composite approach in which the algorithmic knowledge is leveraged by a novel optimal dynamic programming to compute checkpoint dates. We validate these models using a simulator. The model and simulator show that the composite approach drastically increases the performance delivered by an execution platform, especially at scale, by providing the means to increase the interval between checkpoints while simultaneously decreasing the volume of each checkpoint.

*Keywords:* fault-tolerance, resilience, high-performance computing, checkpoint, ABFT, model, performance evaluation

# 1 Introduction

As the processor count increases with each new generation of high performance computing systems, the long dreaded reliability wall is materializing, and threatens to derail the efforts and milestones on the road toward Exascale computing. Despite continuous evolutions, such as improvements to the individual processor reliability, the integration of a large number of components leads to, by simple probabilistic amplification, a stern decrease in the overall capability of High Performance Computing (HPC) platforms to execute long-running applications spanning a large number of resources. Already today, leadership systems encompassing millions of computing elements experience a Mean Time

Between Failures (MTBF) of a few hours [1, 2, 3]. Even considering an optimistic scenario with "fat" nodes, featuring many-core systems and/or GPU accelerators, projections of Exascale machines show unprecedented socket counts which will thereby suffer in terms of reliability due to the sheer number of components used [4].

However, the high performance computing community is not without resources to face this formidable threat. Under the already serious pressure that failures pose to currently deployed systems, checkpointing techniques have seen a large adoption, and many production quality software effectively provide protection against failures with application-level rollback recovery. During the execution, periodic checkpoints are taken that capture the progress of the application. When a failure occurs, the application is terminated, but can later be restarted from the last checkpoint. However, checkpointing techniques inflict severe overhead when failure frequency becomes too high. Checkpoints generate a significant amount of I/O traffic and often block the progression of the application; in addition, they must be taken more and more often as the MTBF decreases in order to enable steady progress of the application. Analytical projections clearly show that sustaining Exascale computing solely with checkpointing will prove challenging [5, 6].

The fault-tolerance community has developed a number of alternative recovery strategies that do not employ checkpoint and rollback recovery as their premise. Strategies such as Algorithm Based Fault Tolerance (ABFT) [7], naturally fault tolerant iterative algorithms [8], resubmission in master-slave applications, etc., can deliver more scalable performance under high stress from process failures. As an example, ABFT protection and recovery activities are not only inexpensive (typically less than 3% overhead observed in experimental works [9, 10]), but also have a negligible asymptotic overhead when increasing node count, which makes them extremely scalable. This is in sharp contrast with checkpointing, which suffers from increasing overhead with system size. ABFT is a useful technique for production systems, offering protection to important infrastructure software such as the dense distributed linear algebra library ScaLAPACK [9]. In the remainder of this paper, without loss of generality, we will use the term ABFT broadly, so as to describe any technique that uses algorithm properties to provide protection and recovery without resorting to rollback recovery.

However, typical HPC applications do spend some time where they perform computations and data movements that are incompatible with ABFT protection. The ABFT technique, as the name indicates, allows for tolerating failures only during the execution of the algorithm that features the ABFT properties. Moreover, it then protects only the part of the user dataset that is managed by the ABFT algorithm. In case of a failure outside the ABFT-protected operation, all data is lost; in case of a failure during the ABFT-protected operation, only the data covered by the ABFT scheme is restored. Unfortunately, these ABFT-incompatible phases force users to resort to general-purpose (presumably checkpoint based) approaches as their sole protection scheme.

Yet, many HPC applications do spend quite a significant part of their total execution time inside a numerical library, and in many cases, these numerical library calls can be effectively protected by ABFT. We believe that the missing link to enable fault tolerance at extreme scale is the ability to effectively compose broad spectrum approaches (such as checkpointing) and algorithm-based recovery techniques, as is most appropriate for different phases within a single application. Possible target applications are based on iterative methods applied across an additional dimension such as time or temperature. Examples of such applications range from heat dissipation to radar cross-section, all of them being extremely time consuming applications, with the usual execution time for real-size problems ranging from several days to weeks. At the core of such applications, a system of linear equations is factorized, and the solution is integrated into a larger context, across the additional dimension. Upon closer inspection of the execution of such an application, it becomes obvious that the most costly step is the factorization of the linear equations. Conveniently, factorization algorithms are some of the first algorithms to be extended with ABFT properties, both in the dense and sparse [11, 12, 13] linear algebra world.

The main contribution of this paper is the design and evaluation of a new composite algorithm that allows for taking advantage of ABFT techniques in applications featuring phases for which no ABFT algorithm exists. We investigate a composition scheme corresponding to the above mentioned type of applications, where the computation alternates between ABFT protected and checkpoint protected phases. This composite algorithm imposes forced checkpoints when entering (and in some cases leaving) library calls that are protected by ABFT techniques, and uses traditional periodic checkpointing, if needed, between these calls. When inside an ABFT-protected call, the algorithm disables all periodic checkpointing. We describe a fault tolerance protocol that enable switching between fault tolerance mechanisms, and depicts how different parts of the dataset are treated at each stage. Based on this scheme, we provide a performance model and use it to predict the expected behavior of such a composite approach on platforms beyond what is currently possible through experimentation. We validate the model by comparing its predicted performance to that obtained with a discrete event simulator.

Another important contribution is the detailed comparison of various checkpointing techniques with ABFT protection, this time not at the whole application level, but rather for a single computational routine (such as a dense matrix LU or QR factorization). The knowledge of the routine characteristics enables us to use incremental checkpointing and a novel dynamic programming approach to optimally place the checkpoints. We compare this technique with classical periodic checkpointing (a la Daly) and the cost of ABFT protection for the routine.

The rest of the paper is organized as follows. We start with a brief overview of related work in Section 2. Then we provide a detailed description of the composite approach in Section 3, and derive the corresponding analytical performance model in Section 4. Section 5 presents a model and a dynamic programming algorithm for incremental checkpointing during computation routines. Section 6 is devoted to evaluating the approach, and comparing the performance of traditional checkpointing protocols with that of the composite approach under realistic scenarios. This comparison is performed both analytically, instantiating the model with the relevant parameters, and in simulation, through an event-based simulator that we specifically designed for this purpose. We obtain an excellent correspondence between the model and the simulations, and we perform a weak-scalability study that demonstrates the full potential of the composite approach at very large scale. We also assess the impact and potential of incremental checkpointing with respect to both classical checkpointing and ABFT protection. Finally, we provide concluding remarks in Section 7.

## 2 Related work

Both hardware and software errors can lead to application failure. The consequence of such errors can take various forms in a distributed system: a definitive crash of some processes, erroneous results or messages, or, at the extreme, corrupted processes exhibiting malignant behavior. In the context of HPC systems, most memory corruptions are captured by ECC memory or similar techniques, leaving process crashes as the most commonly observed type of failures.

The literature is rich in techniques that permit recovering the progress of applications when crash failures strike. The most commonly deployed strategy is checkpointing, in which processes periodically save their state, so that computation can be resumed from that point when some failure disrupts the execution. Checkpointing strategies are numerous, ranging from fully coordinated checkpointing [14] to uncoordinated checkpoint and recovery with message logging [15]. Despite a very broad applicability, all these fault tolerance methods suffer from the intrinsic limitation that both protection and recovery generate an I/O workload that grows with failure probability, and becomes unsustainable at large scale [5, 6] (even when considering optimizations such as diskless or incremental checkpointing [16]).

In contrast, Algorithm Based Fault Tolerance (ABFT) is based on adapting the algorithm so that the application dataset can be recomputed at any moment, without involving costly checkpoints. ABFT was first introduced to deal with silent error in systolic arrays [7]. In recent work, the technique has been employed to recover from process failures [17, 10, 9] in dense and sparse linear algebra factorizations [11, 12, 13], but the idea extends widely to numerous algorithms employed in crucial HPC applications. So called *Naturally Fault Tolerant* algorithms can simply obtain the correct result despite the loss of portions of the dataset (typical of this are master-slave programs, but also iterative refinement methods, like GMRES or CG [8, 18]). Although generally exhibiting excellent performance and resiliency, ABFT requires that the algorithm is innately able to incorporate fault tolerance and therefore stands as a less generalist approach. Another aspect that hinders

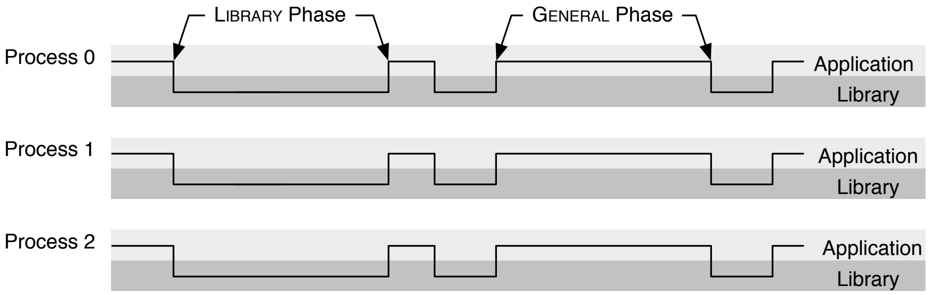


Figure 1: Typical Application

its wide adoption and production deployment is that it can protect *an algorithm* and its dataset, but applications assemble *many* algorithms that operate on different datasets, and which may not all have a readily available ABFT version or employ different ABFT techniques.

To the best of our knowledge, this work is the first to introduce an effective protocol for alternating between generalist (typically checkpoint based) fault tolerance for some parts of the application and custom, tailored techniques (typically ABFT) for crucial, time consuming computational routines.

Many models are available to understand the behavior of checkpoint/restart [19, 20, 21, 22], and thereby to define an optimal checkpoint period. [23] proposes a scalability model to evaluate the impact of failures on application performance. Compared with these works, we include several new key parameters to refine the model. A significant new contribution is to propose a generalized model for a protocol that alternates between checkpointing and ABFT sections. Although most ABFT methods have a complete complexity analysis (in terms of extra-flops, communications incurred by both protection activity and per-recovery cost [10, 9]), modeling the runtime overhead of ABFT methods under failure conditions has never been proposed. The composite model captures both the behavior of checkpointing and ABFT phases, as well as the cost of switching between the two approaches, and thereby permits investing the prospective gain from employing this mixed recovery strategy on extreme scale platforms.

# 3 Composite approach

We consider a typical HPC application whose execution alternates General phases and Library phases (see Figure 1). During General phases, we have no information about the application behavior, and an algorithm-agnostic fault-tolerance technique, like checkpoint and rollback recovery, must be used. On the other hand, during Library phases, we know much more about the application, and we can apply special-purpose fault-tolerance techniques, such as ABFT, to ensure resiliency.

During a General phase, the application can access the whole memory; during a Library phase, only the Library dataset (a subset of the application memory, which is passed as a parameter to the library call) is accessed. The Remainder dataset is the part of the application memory that does not belong to the Library dataset. A strong feature of ABFT is that, in case of failure, the ABFT algorithm can recompute the lost ABFT-protected data based only on the Library dataset of the surviving processors. The major goal of this paper is to compare two fault tolerant approaches:

PurePeriodicCkpt Pure (Coordinated) Periodic Checkpointing refers to the traditional approach based on coordinated checkpoints taken at periodic intervals, and using rollback recovery to recover from failures.

ABFT&PeriodicCkpt Algorithm-Based Fault Tolerance & Periodic Checkpointing refers to the proposed algorithm that combines ABFT techniques in Library phases with Periodic Checkpointing techniques in General phases. It is described below.

Both approaches use PeriodicCkpt techniques, but to a different extent: while PurePeriodicCkpt uses PeriodicCkpt throughout the execution, ABFT&PeriodicCkpt uses it only within General phases of the application.

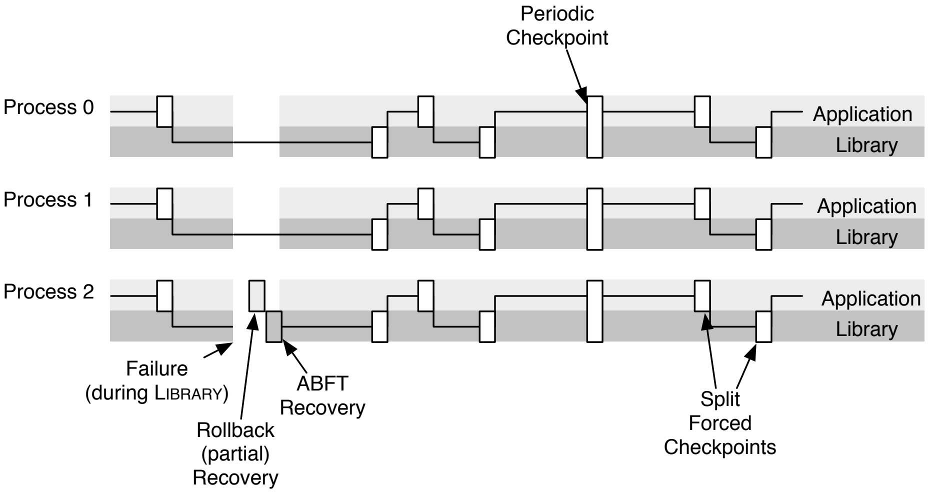


Figure 2: ABFT&PeriodicCkpt composite approach

### 3.1 ABFT&PeriodicCkpt Algorithm

The ABFT&PeriodicCkpt composite approach consists of alternating between periodic checkpointing and rollback recovery on one side, and ABFT on the other side, at different phases of the execution. Every time the application enters a Library phase (that can thus be protected by ABFT), a partial checkpoint is taken to protect the Remainder dataset. The Library dataset, accessed by the ABFT algorithm, need not be saved in that partial checkpoint, since it will be reconstructed by the ABFT algorithm inside the library call.

When the call returns, a partial checkpoint covering the modified Library dataset is added to the partial checkpoint taken at the beginning of the call, to complete it and to allow restarting from the end of the terminating library call. In other words, the combination of the partial entry and exit checkpoints forms a split, but complete, coordinated checkpoint covering the entire dataset of the application.

If a failure is detected while processes are inside the library call, the crashed process is recovered using a combination of rollback recovery and ABFT. ABFT recovery is used to restore the Library dataset before all processes can resume the library call, as would happen with a traditional ABFT algorithm. The partial checkpoint is used to recover the Remainder dataset (everything except the data covered by the current ABFT library call) at the time of the call, and the process stack, thus restoring it before quitting the library routine, see Figure 2. The idea of this strategy is that ABFT recovery will spare some of the time spent redoing work, while periodic checkpointing can be completely de-activated during the library calls.

During General phases, regular periodic coordinated checkpointing is employed to protect against failures. In case of failure, coordinated rollback recovery brings all processes back to the last checkpoint (at most back to the split checkpoint capturing the end of the previous library call).

### 3.2 Efficiency Considerations and Application-Specific Improvements

A critical component to the efficiency of the PeriodicCkpt algorithm is the duration of the checkpointing interval. A short interval increases the algorithm overheads, by introducing many coordinated checkpoints, during which the application experiences slowdown, but also reduces the amount of time lost when there is a failure: the last checkpoint is never long ago, and little time is spent re-executing part of the application. Conversely, a large interval reduces overhead, but increases the time lost in case of failure. The PeriodicCkpt protocol has been extensively studied, and good approximations of the optimal checkpoint interval exist (known as Young and Daly's formula [19, 20]). These approximations are based on the machine MTBF, checkpoint duration, and other parameters. We will consider two forms of PeriodicCkpt algorithms: the PurePeriodicCkpt algorithm,

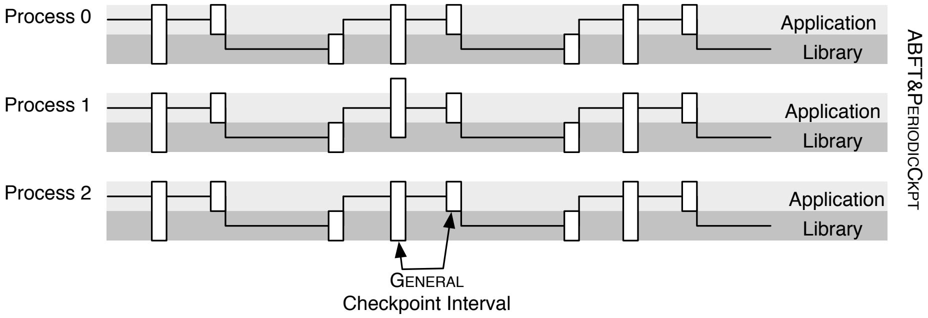


Figure 3: ABFT&PeriodicCkpt composite (time spent in General phases is large)

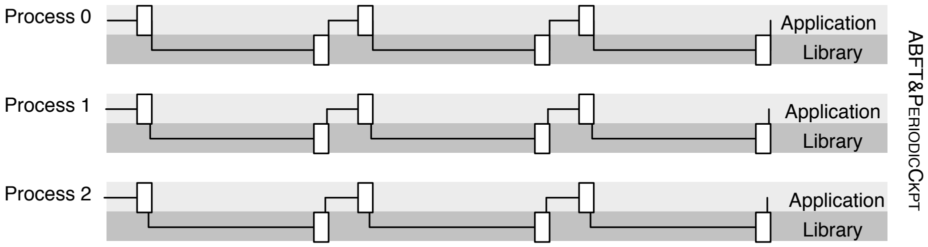


Figure 4: ABFT&PeriodicCkpt composite (time spent in General phases is small)

where a single checkpointing interval is used consistently during the whole execution, and the BiPeriodicCkpt algorithm, where the checkpointing interval may change during the execution, to fit different conditions (see Section 4.3, Figures 5 and 6).

In the ABFT&PeriodicCkpt algorithm, we interleave PeriodicCkpt protected phases with ABFT protected phases, during which periodic checkpointing is de-activated. Thus, different cases have to be considered:

- When the time spent in a General phase is larger than the optimal checkpoint interval, periodic checkpointing is used during these phases in the case of ABFT&PeriodicCkpt (see Figure 3);
- When the time spent in a General phase is smaller than the optimal checkpoint interval, the ABFT&PeriodicCkpt algorithm already creates a complete valid checkpoint for this phase (formed by combining the entry and exit partial checkpoints), so the algorithm will not introduce additional checkpoints (see Figure 4).

Moreover, the ABFT&PeriodicCkpt algorithm forces (partial) checkpoints at the entry and exit of library calls; thus if the time spent in a library call is very small, this approach will introduce more checkpoints than a traditional PeriodicCkpt approach. The time complexity of library algorithms usually depends on a few input parameters related to problem size and resource number, and ABFT techniques have deterministic, well known time overhead complexity. Thus, when possible, the ABFT&PeriodicCkpt algorithm features a safeguard mechanism: if the projected duration of a library call with ABFT protection (computed at runtime thanks to the call parameters and the algorithm complexity) is smaller than the optimal periodic checkpointing interval, then ABFT is not activated, and the corresponding Library phase is protected using the PeriodicCkpt technique only.

Furthermore, since only a subset of the entire dataset is modified during a library call (the Library dataset), application-level checkpointing techniques can benefit PeriodicCkpt approaches. This consists of saving only the subset of the memory that has been modified since the last checkpoint, when taking a new process checkpoint. This influences the duration of the checkpointing operation, and thus the optimal checkpoint interval. In our models, we will take this parameter into consideration.

# 4 Model

In this section, we detail the application model and the various parameters used to quantify the cost of checkpointing and ABFT protection. Then we analytically derive the minimal overhead for all scenarios. In Section 4.1, we start by defining the parameters, and then proceed in Section 4.2 with determining the cost of the composite approach. We compare this cost to that of classical approaches in Section 4.3.

## 4.1 Application and checkpoint parameters

The execution of the application is partitioned into epochs. Within an epoch, there are two phases: the first phase is spent outside the library (it is a General phase), and only periodic checkpointing can be employed to protect from failures during that phase. Then the second phase (a Library phase) is devoted to a library routine that has the potential to be protected by ABFT.

Such a scenario is very general, and many scientific applications obey this scheme, alternating phases spent outside and within a library call that can be protected by ABFT techniques. Since each epoch can be analyzed independently, without loss of generality, we focus on a single epoch. Let us introduce some notations. The total duration of the epoch is $T_0 = T_G + T_L$, where $T_G$ and $T_L$ are the durations for the General and Library phases, respectively. Let $\alpha$ be the fraction of time spent in a Library phase: then we have $T_L = \alpha \times T_0$ and $T_G = (1 - \alpha) \times T_0$.

As mentioned earlier, another important parameter is the amount of memory that is accessed during the Library phase (the Library dataset). This parameter is important because the cost of checkpointing in each phase is directly related to the amount of memory that needs to be protected. The total memory footprint is $M$, and the associated checkpointing cost is $C$ (we assume a finite checkpointing bandwidth, so $C > 0$). We write $M = M_L + M_{\overline{L}}$, where $M_L$ is the size of the Library dataset, and $M_{\overline{L}}$ is the size of the Remainder dataset. Similarly, we write $C = C_L + C_{\overline{L}}$, where $C_L$ is the cost of checkpointing $M_L$, and $C_{\overline{L}}$ the cost of checkpointing $M_{\overline{L}}$. We can define the parameter $\rho$ that defines the relative fraction of memory accessed during the Library phase by $M_L = \rho M$, or, equivalently, by $C_L = \rho C$.

## 4.2 Cost of the composite approach

We now detail the cost of resilience during each phase of the composite approach. We start with the intrinsic cost of the method itself, i.e., assuming a fault-free execution. Then we account for the cost of failures and recovery.

### 4.2.1 Fault-free execution

During the General phase, we separate two cases. First, if the duration $T_G$ of this phase is short, *i.e.* smaller than $P_G$-$C_{\overline{L}}$, which is the amount of work during one period of length $P_G$ (and where $P_G$ is determined below), then we simply take a partial checkpoint at the end of this phase, before entering the ABFT-protected mode. This checkpoint is of duration $C_{\overline{L}}$, because we need to save only the Remainder dataset in this case. Otherwise, if $T_G$ is larger than $P_G$-$C_{\overline{L}}$, we rely on periodic checkpointing during the General phase: more specifically, the regular execution is divided into periods of duration $P_G = W + C$. Here $W$ is the amount of work done per period, and the duration of each periodic checkpoint is $C = C_{\overline{L}} + C_L$, because the whole application footprint must be saved during a General phase. The last period is different: we execute the remainder of the work, and take a final checkpoint of duration $C_{\overline{L}}$ before switching to ABFT-protected mode. The optimal (approximated) value of $P_G$ will be computed below.

Altogether, the length $T_G^{\text{ff}}$ of a fault-free execution of the General phase is the following:

- If $T_G \leq P_G - C_{\overline{L}}$, then $T_G^{\text{ff}} = T_G + C_{\overline{L}}$
- Otherwise, we have $\lfloor \frac{T_G}{W} \rfloor$ periods of length $P_G$, plus possibly a shorter last period if $T_G$ is not evenly divisible by $W$. In addition, we need to remember that the last checkpoint taken is of length $C_{\overline{L}}$ instead of $C$.

This leads to

$$T_G^{\text{ff}} = \begin{cases} T_G + C_{\overline{L}} & \text{if } T_G \leq P_G - C_{\overline{L}} \\ \lfloor \frac{T_G}{P_G - C} \times P_G \rfloor + (T_G \bmod W) + C_{\overline{L}} & \text{if } T_G > P_G - C_{\overline{L}} \text{ and } T_G \bmod W \neq 0 \\ \frac{T_G}{P_G - C} \times P_G - C_L & \text{if } T_G > P_G - C_{\overline{L}} \text{ and } T_G \bmod W = 0 \end{cases} \tag{1}$$

Now consider the Library phase: we use the ABFT-protection algorithm, whose cost is modeled as an affine function of the time spent: if the computation time of the library routine is $t$, its execution with the ABFT-protection algorithm becomes $\phi \times t$. Here, $\phi > 1$ accounts for the overhead paid per time-unit in ABFT-protected mode. This linear model for the ABFT overhead fits the existing algorithms for linear algebra, but other models could be considered. In addition, we pay a checkpoint $C_L$ when exiting the library call (to save the final result of the ABFT phase). Therefore, the fault-tree execution time is

$$T_L^{\text{ff}} = \phi \times T_L + C_L \tag{2}$$

Finally, the fault-free execution time of the whole epoch is

$$T^{\text{ff}} = T_G^{\text{ff}} + T_L^{\text{ff}} \tag{3}$$

where $T_G^{\text{ff}}$ and $T_L^{\text{ff}}$ are computed according to the Equations (1) and (2).

### 4.2.2 Cost of failures

Next we have to account for failures. During $t$ time units of execution, the expectation of the number of failures is $\frac{t}{\mu}$, where $\mu$ is the Mean Time Between Failures on the platform. We start with a discussion on the definition of $\mu$. Consider a platform comprising $N$ identical resources, whose individual failure inter-arrival times $X_i^{(j)}$ are I.I.D. (Independent Identically Distributed) random variables. Here, $X_i^{(j)}$ is the time elapsed between the occurrence of the $j-1$-th failure on node $i$ (or the beginning of the execution if $j = 1$), and the occurrence of the $j$-th failure on the same node $i$. All $X_i^{(j)}$ follow the same probability distribution, whose expectation is $\mu_{\text{ind}}$, the MTBF on each node. Now consider the whole platform, and let $Y^{(j)}$ denote the failure inter-arrival times on the platform (for instance $Y^{(1)}$, the time until the first failure, is the minimum of the $X_i^{(1)}$, $1 \leq i \leq N$). Unfortunately, the $Y^{(j)}$ are not I.I.D., unless the $X_i^{(j)}$ follow an Exponential distribution, so they do not have the same expectation in the general case. In the literature, there are two (equivalent) alternative definitions of $\mu$, the MTBF of the platform, and we briefly sketch both of them.

The first alternative to define $\mu$ is based on the expected total number of faults. Let $n(t)$ be the expectation of the total number of failures on the whole platform from time 0 to time $t$, and define the platform MTBF $\mu$ as the limit ratio of time over failure number:

$$\mu = \lim_{t \to +\infty} \frac{t}{n(t)}$$

By definition, $n(t) = \sum_{i=1}^{N} n_i(t)$, where $n_i(t)$ is the expected number of failures on node $i$ until time $t$. Using Wald's law (and the fact that the $X_i^{(j)}$ are I.I.D.), it is shown in [24] that $\mu_{\text{ind}} = \lim_{t \to +\infty} \frac{t}{n_i(t)}$ for $1 \leq i \leq N$, hence that $\mu = \frac{\mu_{\text{ind}}}{N}$.

The second alternative to define $\mu$ is based on the average value of the $Y_i$, using the theory of the superposition of renewal processes [25]: let us now define

$$\mu = \lim_{n \to +\infty} \frac{\sum_{i=1}^{n} \mathbb{E}\left(Y_i\right)}{n}$$

Then Kella and Stadje [26, Theorem 4] proves that this limit indeed exists and is also equal to $\frac{\mu_{\text{ind}}}{N}$, as soon as the distribution function of the $X_i^{(j)}$ is continuous.

In summary, we use the relation $\mu = \frac{\mu_{\text{ind}}}{N}$ with either definition. This relation is agnostic of the granularity of the resources, which can be anything from a single CPU to a complex multi-core socket.

We are ready to compute the cost of failures in both execution phases. For each phase, we have a similar equation: the final execution time is the fault-free execution time, plus the number of failures multiplied by the (average) time lost per failure:

$$T_G^{\text{final}} = T_G^{\text{ff}} + \frac{T_G^{\text{final}}}{\mu} \times t_G^{\text{lost}} \tag{4}$$

$$T_L^{\text{final}} = T_L^{\text{ff}} + \frac{T_L^{\text{final}}}{\mu} \times t_L^{\text{lost}} \tag{5}$$

Equation (4) reads as follows: $T_G^{\text{ff}}$ is the failure-free execution time, to which we add the time lost due to failures; the expected number of failures is $\frac{T_G^{\text{final}}}{\mu}$, and $t_G^{\text{lost}}$ is the average time lost per failure. We have a similar reasoning for Equation (5). Then, $t_G^{\text{lost}}$ and $t_L^{\text{lost}}$ remain to be computed.

For $t_G^{\text{lost}}$ (General phase), we discuss both cases:

- If $T_G \leq P_G - C_{\overline{L}}$: since we have no checkpoint until the end of the General phase, we have to redo the execution from the beginning of the phase. On average, the failure strikes at the middle of the phase, hence the expectation of loss is $\frac{T_G^{\text{ff}}}{2}$ time units. We then add the downtime $D$ (time to reboot the resource or set up a spare) and the recovery $R$. Here $R$ is the time needed for a complete reload from the checkpoint (and $R = C$ if read/write operations from/to the stable storage have the same speed). We derive that:

$$t_G^{\text{lost}} = D + R + \frac{T_G^{\text{ff}}}{2} \tag{6}$$

- If $T_G > P_G - C_{\overline{L}}$: in this case, we have periodic checkpoints, and the amount of execution which needs to be re-done after a failure corresponds to half a checkpoint period on average, so that:

$$t_G^{\text{lost}} = D + R + \frac{P_G}{2} \tag{7}$$

For $t_L^{\text{lost}}$ (Library phase), we derive that

$$t_L^{\text{lost}} = D + R_{\overline{L}} + \text{Recons}_{\text{ABFT}}$$

Here, $R_{\overline{L}}$ is the time for reloading the checkpoint of the Remainder dataset (and in many cases $R_{\overline{L}} = C_{\overline{L}}$). As for the Library dataset, there is no checkpoint to retrieve, but instead it must be reconstructed from the ABFT checksums, which takes time $\text{Recons}_{\text{ABFT}}$.

### 4.2.3 Optimization

We verify from Equations (2) and (5) that $T_L^{\text{final}}$ is always a constant. Indeed, we derive that:

$$T_L^{\text{final}} = \frac{1}{1 - \frac{D + R_{\overline{L}} + \text{Recons}_{\text{ABFT}}}{\mu}} \times (\phi \times T_L + C_L) \tag{8}$$

As for $T_G^{\text{final}}$, it depends on the value of $T_G$: it is constant when $T_G$ is small. In that case, we derive that:

$$T_G^{\text{final}} = \frac{1}{1 - \frac{D + R + \frac{T_G + C_{\overline{L}}}{2}}{\mu}} \times (T_G + C_{\overline{L}}) \tag{9}$$

The interesting case is when $T_G$ is large: in that case, we have to determine the optimal value of the checkpointing period $P_G$ which minimizes $T_G^{\text{final}}$. We use an approximation here: we assume that we have an integer number of periods, and the last periodic checkpoint is of size $C$. Note that the larger $T_G$, the more accurate the approximation. From Equations (1), (4) and (7), we derive the following simplified expression:

$$T_G^{\text{final}} = \frac{T_G}{X} \text{ where } X = \left(1 - \frac{C}{P_G}\right)\left(1 - \frac{D + R + \frac{P_G}{2}}{\mu}\right) \tag{10}$$

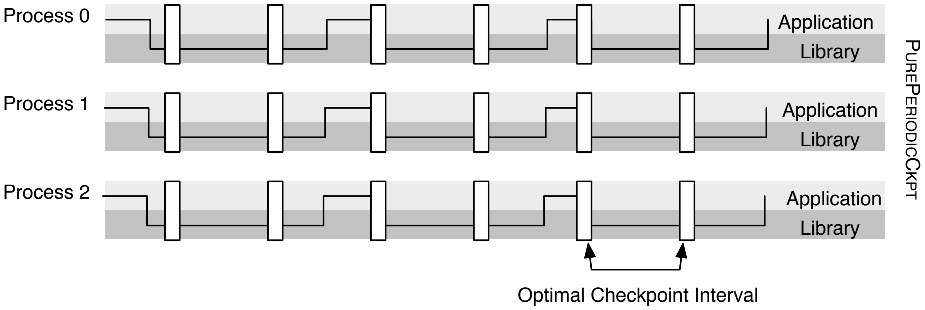


Figure 5: PurePeriodicCkpt

We rewrite:

$$X = (1 - \frac{C}{2\mu}) - \frac{P_G}{2\mu} - \frac{C(\mu - D - R)}{\mu P_G}$$

The maximum of $X$ gives the optimal period $P_G^{\text{opt}}$. Differentiating $X$ as a function of $P_G$, we find that it is obtained for:

$$P_G^{\text{opt}} = \sqrt{2C(\mu - D - R)} \tag{11}$$

Plugging the value of $P_G^{\text{opt}}$ back into Equation (10) provides the optimal value of $T_G^{\text{final}}$ when $T_G$ is large.

We have successfully computed the final execution time $T^{\text{final}}$ of our composite approach in all cases. In the experiments provided in Section 6, we report the corresponding *waste*. The waste is defined as the fraction of time when platform resources do not progress the application's computation (due to the intrinsic overhead of the resilience technique and to failures that strike the application during execution). The waste is given by:

$$\textsc{Waste} = 1 - \frac{T_0}{T^{\text{final}}} \tag{12}$$

We conclude this section with another word of caution: the optimal value of the waste is only a first-order approximation, not an exact value. Equation (11) is a refined version of well known formulas by Young [19] and Daly [20]. But just as in [19, 20], the formula only holds when $\mu$, the value of the MTBF, is large with respect to the other resilience parameters. Owing to this hypothesis, we can neglect the probability of several failures occurring during the same checkpointing period. However, in order to assess the accuracy of the model, when doing simulations in section 6, we account for all unlikely failure scenarios, including multiple rollbacks during a period, and re-execute the work until each period is indeed successfully completed. Also, we use arbitrary values for $T_G$, not just multiples of the optimal period $P_G^{\text{opt}}$, and always conclude a General phase by a checkpoint $C_{\overline{L}}$.

## 4.3 Comparison with conservative approaches

A fully conservative approach, agnostic of the ABFT library, would perform periodic checkpoints throughout the execution of the whole epoch. As already mentioned, we call this approach PurePeriodicCkpt (see Figure 5). Let $T_{PC}^{\text{final}}$ be the final execution time with this PurePeriodicCkpt approach; it can be computed from the results of Section 4.2 as follows:

- No ABFT: $\alpha = 0$ and $T_L^{\text{final}} = 0$
- We optimize $T_{PC}^{\text{final}} = T_G^{\text{final}}$ just as before, with the same optimal period $P_{PC}^{\text{opt}} = P_G^{\text{opt}}$, employed throughout the epoch.

One can reduce the cost of PurePeriodicCkpt by noticing that during the Library epoch, only the Library dataset is modified (namely $M_L$). Employing application-level checkpointing would, in this case, yield a checkpoint cost reduction (down to $C_L$). Obviously, with a different cost of checkpointing, the optimal checkpoint period is different. Therefore, a *semi-conservative approach*

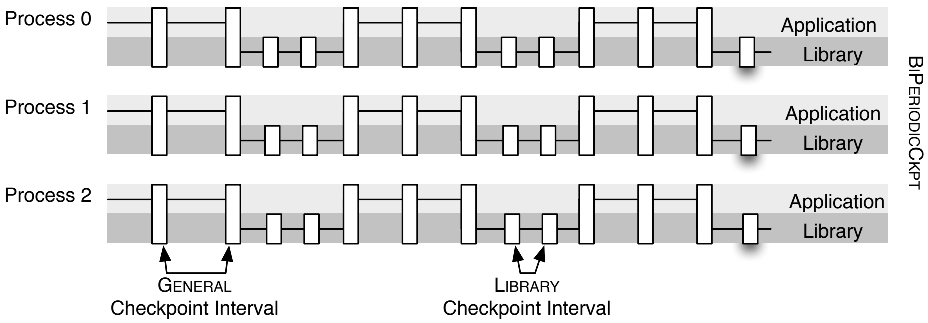


Figure 6: BiPeriodicCkpt

(called BiPeriodicCkpt, see Figure 6) assumes that the checkpoint system can recognize that the program has entered a library routine that modifies only a subset of the dataset, and switches to the optimal checkpoint period according to the application phase. During the General phase, the overhead of failures and protection remains unchanged, but during the Library phase, the cost of a checkpoint is reduced to $C_L$ (instead of $C$); however, the cost of reloading from a checkpoint remains $R$ (since the different application-level checkpoints must be combined to recover the entire dataset at rollback time). This leads to two different checkpointing periods, one for each phase. The new optimal checkpoint period can be modeled as follows:

- $T_{PC}^{\text{final}} = T_G^{\text{final}} + T_{LPC}^{\text{final}}$, where $T_G^{\text{final}}$ is computed as before
- $T_{LPC}^{\text{final}}$ is computed similarly as $T_G^{\text{final}}$, but with different parameters:

$$T_{LPC}^{\text{final}} = \frac{1}{1 - \frac{D+R+\frac{P_{BPC}}{2}}{\mu}} \times \frac{P_{BPC}}{P_{BPC} - C_L} \times T_L \tag{13}$$

and the optimal period is

$$P_{BPC,L}^{\text{opt}} = \sqrt{2C_L(\mu - D - R)} \tag{14}$$

# 5 Incremental checkpointing

In this section, we refine our evaluation of checkpointing vs. ABFT during the Library phase. In addition to comparing ABFT with (classical) periodic checkpointing, we also compare it with *incremental checkpointing* applied during the Library routines. This technique relies on knowledge about the computational routine: during the execution of the routine, the overhead of checkpointing can be reduced owing to a careful selection of the time-steps at which checkpoints are taken, in order to minimize the volume of data that needs to be saved during these checkpoints.

## 5.1 Application model

We target a typical library routine that consists of several consecutive tasks. Each task is executed in parallel on the platform, and can be followed by a checkpoint. To give an example, a typical task could be factoring a matrix panel within the HPL benchmark [27]. However, our approach is agnostic of the granularity of the tasks, and a finer granularity can always be achieved by subdividing the routine workload into more tasks.

Formally, the library routine is partitioned into $n$ tasks $\mathrm{T}_1 \ldots \mathrm{T}_n$. The execution time of $\mathrm{T}_i$ on the platform is $\mathrm{W}_i$. Task $\mathrm{T}_i$ may be followed by a checkpoint, whose cost is incremental: it depends on the number of memory locations cumulatively modified by all the tasks executed since the last checkpoint. $\mathrm{C}_{i\ldots j}$ represents the cost of checkpointing after task $\mathrm{T}_j$, when the last checkpoint precedes $\mathrm{T}_i$. Refer to Table 1 for a summary of the notations.

When a failure strikes, we have to re-execute all tasks that follow the last checkpoint. The cost of reloading from the checkpoint preceding $\mathrm{T}_i$ is $\mathrm{R}_{i-1}$. The cost of restarting from the initial state

| | |
|---|---|
| $\mathrm{T}_i$ | an individual task that represents a checkpointable unit |
| $\mathrm{W}_i$ | duration of work for task $\mathrm{T}_i$ |
| $\mathrm{W}_{i\ldots j}$ | cumulative duration of work for tasks $\mathrm{T}_i \ldots \mathrm{T}_j$ |
| $\mathrm{C}_{i\ldots j}$ | duration of checkpoint after $\mathrm{T}_j$, knowing that last checkpoint is after $\mathrm{T}_{i-1}$ |
| $\mathrm{R}_i$ | duration of restart from checkpoint after $\mathrm{T}_i$ |
| $\tau_{io}$ | collective I/O transfer throughput |
| $\tau_{flops}$ | collective computation throughput |
| $\lambda$ | $1/\mu$ (rate of Exponentially-distributed failures) |

Table 1: List of model parameters.

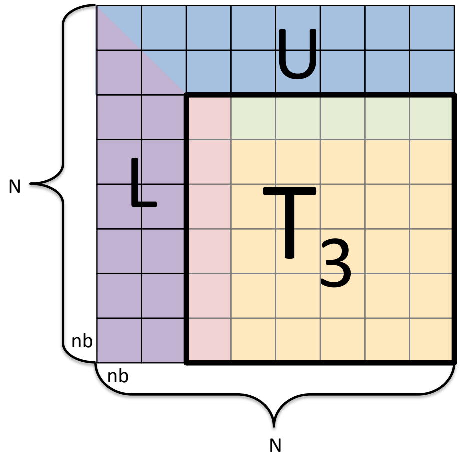


Figure 7: Representation of a tiled LU Factorization. Once the L and U parts of the matrix have been computed, they are not touched by the algorithm before the end of the factorization. The figure shows the part of the matrix that is updated by Task $\mathrm{T}_3$.

is $\mathrm{R}_0$. Note that except for $\mathrm{T}_1$, it is possible to restart at $\mathrm{T}_i$ only when a checkpoint was taken after $\mathrm{T}_{i-1}$ (therefore, $\mathrm{R}_{i-1}$ does not include the cost of computing any preceding task). The cost of $\mathrm{R}_{i-1}$ is fixed for a given value of $i$ and does not depend on the number of tasks separating two checkpoints.

To illustrate these definitions, consider the dense LU or QR factorization of a square matrix of size $N$. This matrix is partitioned into square tiles of size $n_b$, and there are $n \times n$ tiles (so that $N = n \times n_b$). We partition the factorization into $n$ tasks, where $\mathrm{T}_i$ corresponds to the $i$-th step of the factorization: factor panel $i$ (the $i$-column of the tiled matrix) and update columns $i+1$ to $n$. For an LU factorization (see Figure 7), only the tiles on, or below the diagonal are modified (we ignore pivoting for simplification, as this does not impact the checkpoint size), so that $\mathrm{T}_i$ operates on the bottom-right corner of tiles, those of index $i$ to $n$. We derive that $\mathrm{C}_{i\ldots j}$ depends on $i$ but not on $j$ and corresponds to the cost of writing this bottom-right corner onto stable storage. There are $(n-i+1)^2$ tiles of $n_b^2$ elements, hence $\mathrm{C}_{i\ldots j} = (n-i+1)^2 n_b^2 \tau_{io}$, where $\tau_{io}$ is the time to write a floating point number to stable storage. We similarly derive that $\mathrm{R}_{i-1} = \mathrm{C}_{i\ldots j}$, assuming the same speed for reading from and writing to stable storage. Finally, for the QR factorization, we obtain the same checkpoint and recovery costs as for LU, however, the computational cost is different. For LU, task $\mathrm{T}_i$ requires $2(n-i+1)^2 n_b^3$ flops, hence an execution time $\mathrm{W}_i = 2(n-i+1)^2 n_b^3 \tau_{flops}$, where $\tau_{flops}$ is the (parallel) time to execute a double-precision floating-point operation. For QR, the load of each task is twice as high, and the computation rate is typically slower, because QR kernels are not as efficient as LU kernels (employing LARFB kernels instead of GEMM matrix products for the updates).

We can envision more complex applications for which $\mathrm{C}_{i\ldots j}$ would depend on both $i$ and $j$. An example is the *left-looking* LU factorization [28], where at step $i$, only the $i$-th panel is updated. But in this variant, the $i$-th panel is updated $i-1$ times at step $i$, receiving the updates from all the previous steps. In that case, we have $\mathrm{C}_{i\dots j} = (j-i+1)^2 n_b^2 \tau_{io}$, because only panels $i$ to $j$ have been modified between $\mathrm{T}_i$ and $\mathrm{T}_j$. On average, the checkpoint time is reduced in comparison with the previous (so-called right-looking) variant. However, we need the entire matrix to restart the execution at any step, hence $\mathrm{R}_{i-1} = n^2 n_b^2 \tau_{io}$. Also, while the flop counts of both versions are the same, the parallel efficiency of the left-looking version is not as good as that of the right-looking one, because a smaller fraction of the matrix (a single panel instead of the whole trailing matrix) is updated at each step. In other words, we would use a larger value of $\tau_{flops}$ for the left-looking version.

The optimization problem can be stated as follows:

**Definition 1** (Incremental)**.** *Given $n$ consecutive tasks $\mathrm{T}_i$ of execution time $\mathrm{W}_i$, $1 \leq i \leq n$, given the values $\mathrm{C}_{i\dots j}$, $1 \leq i \leq j \leq n$ and $\mathrm{R}_i$, $0 \leq i \leq n-1$, and given the failure MTBF $\mu$, decide after which tasks to insert checkpoints so as to minimize the expectation of the total execution time.*

We provide an optimal solution for Incremental when the failure inter-arrival times follow an Exponential distribution (of rate $\lambda = 1/\mu$) below. For an arbitrary distribution, one can derive an approximation by using the result obtained with an Exponential distribution for the same MTBF.

## 5.2 Optimal incremental checkpointing strategy

This section provides a dynamic programming algorithm to solve the Incremental problem. We use a dynamic programming algorithm to compute $Opt(i,n)$ for $1 \leq i \leq n$, the optimal runtime expectation for executing all tasks $\mathrm{T}_i$, $\mathrm{T}_{i+1}$, …, $\mathrm{T}_n$, knowing that a checkpoint has been taken just after task $\mathrm{T}_{i-1}$ and that one checkpoint will be taken after task $\mathrm{T}_n$. We assume that the application data is originally stored on disk, hence $C_0 = 0$ where $C_0$ is the time needed to prepare for $\mathrm{R}_0$. We also assume that it will be stored on disk at the end of the execution, hence the need to take a checkpoint after the last task. In fact we are interested only in the value of $Opt(1,n)$, but we will compute it recursively from the other values $Opt(i,n)$ ($1 \leq i \leq n$), and from the expected time needed to successfully execute some amount of work and checkpoint it. We start with the latter: let $f(i,j)$ denote the expected time to successfully execute tasks $\mathrm{T}_i$, $\mathrm{T}_{i+1}$, …, $\mathrm{T}_j$, and to checkpoint after $\mathrm{T}_j$, without any intermediate checkpoint, and knowing that a checkpoint has been taken after task $\mathrm{T}_{i-1}$. To the best of our knowledge, the expectation $E(W,C)$ of the time needed to successfully compute during $W$ seconds and then take a checkpoint of duration $C$ is known only for Exponentially distributed failures; from [22], we know that:

$$E(W,C) = e^{\lambda R}(\frac{1}{\lambda} + D)(e^{\lambda(W+C)} - 1)$$

where $\lambda$ is the failure rate (inverse of MTBF $\mu$). We readily obtain:

$$f(i,j) = e^{\lambda \mathrm{R}_{i-1}}(\frac{1}{\lambda} + D)(e^{\lambda(\mathrm{W}_{i\dots j} + \mathrm{C}_{i\dots j})} - 1)$$

The value of $Opt(i,n)$ can be computed using the following recursive dynamic programming formula, where $1 \leq i < n$:

$$Opt(i,n) = \min\left\{\ f(i,n), \quad \min_{i \leq h < n} f(i,h) + Opt(h+1,n)\ \right\} \tag{15}$$

The formula can be understood as follows: we search for all possible positions $h$ of the first checkpoint after task $\mathrm{T}_i$, recursively using the optimal solution for the subproblem $Opt(h+1,n)$, and we take the minimum expected execution time over all these values. We also account for the possibility of no checkpoint until the last task $\mathrm{T}_n$ –hence the value $f(i,n)$. The formula is initialized by letting $Opt(n,n) = f(n,n)$.

This dynamic programming formulation nicely extends the result of Toueg [29], who has a similar approach for deciding which tasks to checkpoint in a linear chain of tasks. The main difference is that the checkpoint time of tack $\mathrm{T}_j$ in [29] is independent of the location of the last checkpoint ($\mathrm{C}_{i\ldots j} = C_j$ for all $i$).

For a general HPC application, let Inc&PeriodicCkpt denote the approach that uses optimal incremental checkpointing in the Library phase, and regular checkpointing in the General phase.

# 6 Evaluation

In this section, we evaluate the ABFT&PeriodicCkpt protocol in simulation, and compare its performance to PurePeriodicCkpt, BiPeriodicCkpt and Inc&PeriodicCkpt in different scenarios. We start with a description of the simulator and experiments in Section 6.1. Then we detail the results of the comparison of the different protocols in Section 6.2. In Section 6.2, we also compare simulation results and predicted performance results analytically computed from the models presented in Sections 4.2 and 4.3, and we do obtain a very good correspondence. Then, we conduct a weak scalability study in Section 6.3, in order to assess the performance of the various protocols at very large scale. Last, in Section 6.4, we investigate the potential of Inc&PeriodicCkpt, where incremental checkpointing is employed during the Library phases.

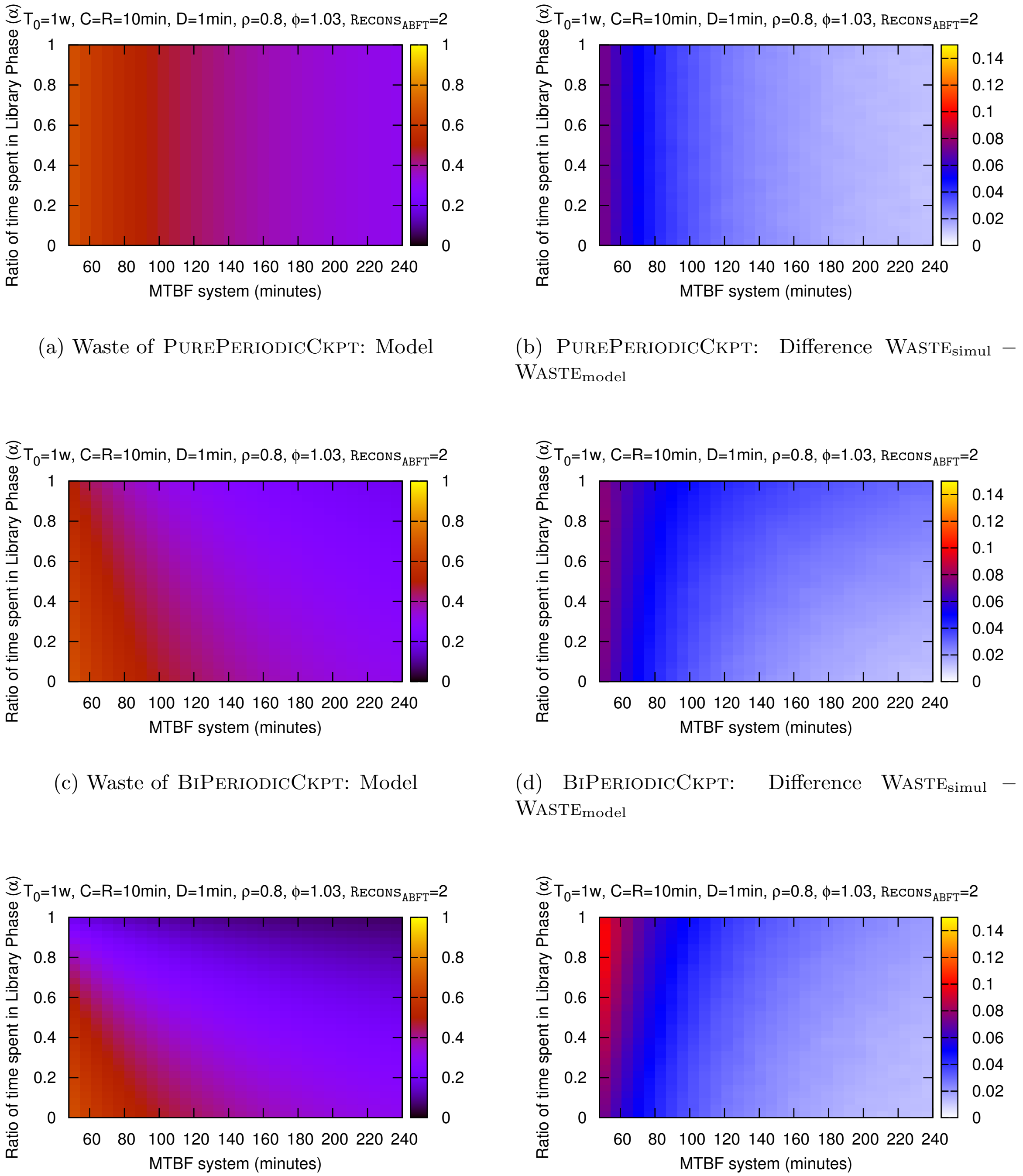
## 6.1 Validation

To validate the performance models, we have implemented a simulator, based on discrete event simulation, that reproduces the behavior of the different algorithms, even in cases that the performance models cannot cover. Indeed, as mentioned in Section 4.2.3, a few approximations have been made when considering the mathematical models, to make their expressions tractable. For example, the models assume that a single failure may hit the system, until its recovery. The effect of events like overlapping failures, which is uncommon when the MTBF is large enough, is neglected in the proposed performance model. The simulator, however, takes these events into account, accurately reproducing the corresponding costs.

In the simulator, failures are generated following an Exponential distribution law parameterized to fix the MTBF to a given value. Then the application, and the chosen fault tolerance mechanism, are unfolded on that set of failures, triggering rollbacks, and other protocol-specific overheads, to measure the duration of the execution. For each scenario, and each parameter, the average termination time over a thousand executions is returned by the simulator.

We present in [30] , an exhaustive evaluation of the different parameters independently, comparing the results as predicted by the models, and the simulation. In this paper, we focus the analysis on a smaller subset. We consider an application that executes for a week when there is neither a fault tolerance mechanism nor any failure. The time required to take a checkpoint and rollback the whole application is 10 minutes ($C$, $R$), a consistent order of magnitude for current applications at large scale [5]. We consider that the ratio of the memory that is modified by the Library phase ($\rho$) is fixed at 0.8 (to vary a single parameter at a time in our simulation), and the overhead due to ABFT is $\phi = 1.03$ (again, typical from production deployments [9]).

Figure 8 presents 6 evaluations of that scenario. The MTBF of the system varies on the x-axis, and the ratio of time spent in the Library phase ($\alpha$) on the y-axis. In Figures 8a to 8f, we present the waste predicted by the model, and validate the model by observing the difference between the model prediction and the waste measured from the simulator for a given combination of parameters and protocol. From the validation perspective, the figures on the right side show an excellent correspondence between predicted (from the model) and actual (obtained from simulation) values. For small MTBF values, the model tends to slightly underestimate the waste. Qualitatively, this under-estimation is expected, because an approximation that must be done to allow a closed formula representation is to assume that failures will not hit processors while they are recovering from a previous failure. In reality, when the MTBF is very small, this event can sometimes happen, forcing the system to start a new recovery, and introducing additional waste. That underestimation does not exceed 12% in the worst case and quickly decreases to below 5%.



(a) Waste of PUREPERIODICCKPT: Model

(b) PUREPERIODICCKPT: Difference $\text{WASTE}_{\text{simul}} - \text{WASTE}_{\text{model}}$

(c) Waste of BIPERIODICCKPT: Model

(d) BIPERIODICCKPT: Difference $\text{WASTE}_{\text{simul}} - \text{WASTE}_{\text{model}}$

(e) Waste of ABFT&PERIODICCKPT: Model

(f) ABFT&PERIODICCKPT: Difference $\text{WASTE}_{\text{simul}} - \text{WASTE}_{\text{model}}$

Figure 8: Waste as a function of MTBF and fraction of time $\alpha$ spent in LIBRARY phase; difference of the measured waste by simulation $\text{WASTE}_{\text{simul}}$ minus the predicted waste by the model $\text{WASTE}_{\text{model}}$. Here $W$ =1 week, $C$ =$R$ =10 minutes, $C_L$ =0.8$C$, $\phi$ =1.03, $\text{Recons}_{\text{ABFT}}$ =2s.

## 6.2 PurePeriodicCkpt, BiPeriodicCkpt, and ABFT&PeriodicCkpt

Consider Figures 8a and 8b, that represent the waste of PurePeriodicCkpt as a function of the MTBF ($\mu$) and the amount of time spent in the Library routine ($\alpha$): it is obvious that the PurePeriodicCkpt protocol, which is oblivious of the different phases of the application, presents a waste that is only a function of the MTBF. As already evaluated and explained in many other works, when the MTBF increases, the waste decreases, because the overheads due to failure handling tend toward 0, and the optimal checkpointing period can increase significantly, reducing the waste due to resilience in a fault-free execution.

Comparatively, for the protocol BiPeriodicCkpt presented in Figures 8c and 8d, the parameter $\alpha$ affects the optimal periods used both in the Library and general phases. Since the cost of checkpointing for these phases differs by 20% ($C_L = 0.8C$), when the relative time spent in the General routine increases ($\alpha$ is closer to 0), then the protocol behaves more and more as PurePeriodicCkpt. When $\alpha$ is almost 1, on the other hand, the behavior is similar to PurePeriodicCkpt, but with a checkpoint cost reduction of 20%. Thus, the waste becomes minimal when $\alpha$ tends toward 1.

In Figures 8e and 8f, we present the waste for the ABFT&PeriodicCkpt protocol. When $\alpha$ tends toward 0, as above, the protocol behaves as PurePeriodicCkpt, and no benefit is shown. When 50% of the time is spent in the Library routine, the benefit, compared to PurePeriodicCkpt, but also compared to BiPeriodicCkpt, is already visible: for 50% of the failures (when the failure hits during a Library phase), the cost of recovery is reduced to 20% of the rollback cost, plus the constant overhead of ABFT recovery. Moreover, periodic checkpointing is disabled 50% of the time, producing yet another gain compared to BiPeriodicCkpt which still requires saving 80% of the memory periodically. In this case, the gain in checkpoint avoidance compensates for the waste induced by additional computations done during the Library phase to provide the ABFT protection. When considering the extreme case of 100% of the time spent in the Library phases, the overhead tends to reach the overhead induced by the slowdown factor of ABFT ($\phi = 1.03$, hence 3% overhead).

## 6.3 Weak Scalability

As illustrated above, the ABFT&PeriodicCkpt approach exhibits better performance when a significant time is spent in the Library phase, *and* when the failure rate implies a small optimal checkpointing period. If the checkpointing period is large (because failures are rare), or if the duration of the Library phase is small, then the optimal checkpointing interval becomes larger than the duration of the Library phase, and the algorithm automatically resorts to the BiPeriodicCkpt protocol. This can also be the case when the epoch itself is smaller than (or of the same order of magnitude as) the optimal checkpointing interval (i.e., when the application does a fast switching between Library and General phases).

However, consider such an application that frequently switches between (relatively short) Library and General phases. When porting that application to a future larger scale machine, the number of nodes that are involved in the execution will increase, and at the same time, the amount of memory on which the ABFT operation is applied will grow (following Gustafson's law). This has a double impact: the time spent in the ABFT routine increases, while at the same time, the MTBF of the machine decreases. In this section, we evaluate quantitatively how this scaling factor impacts the relative performance of the ABFT&PeriodicCkpt, PurePeriodicCkpt and BiPeriodicCkpt algorithms. Owing to the good correspondence between results from the model and results from the simulation, we (confidently) use only the model in this scalability study.

First, we consider the case of an application where the Library and General phases scale at the same rate. We take the example of linear algebra kernels operating on $2D$-arrays (matrices), that scale in $O(n^3)$ of the array order $n$ (in both phases). Following a weak scaling approach, the application uses a fixed amount of memory $M_{ind}$ per node, and when increasing the number $x$ of nodes, the total amount of memory increases linearly as $M = xM_{ind}$. Thus $O(n^2) = O(x)$, and the parallel completion time of the $O(n^3)$ operations, assuming perfect parallelism, scales in $O(\sqrt{x})$.

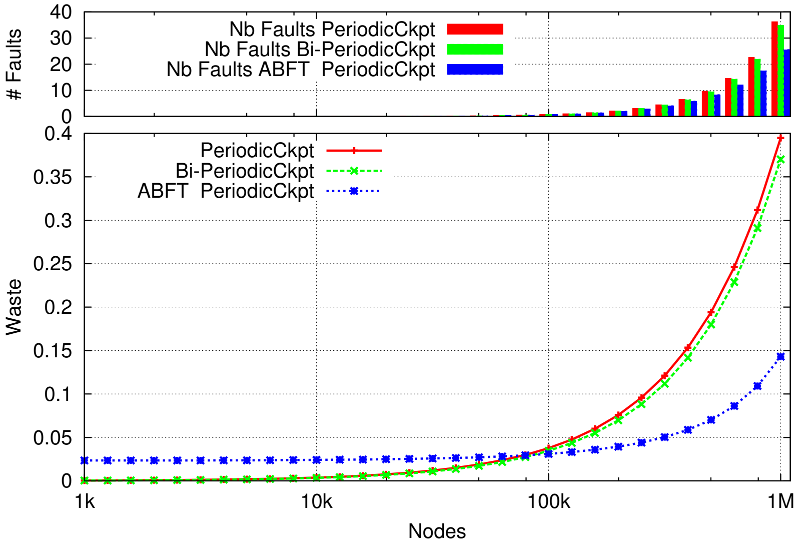


Figure 9: Total waste for ABFT&PeriodicCkpt, BiPeriodicCkpt and PurePeriodicCkpt, when considering the weak scaling of an application with a fixed ratio of 80% spent in a Library routine.

To instantiate this case, we take an application that would last a thousand minutes at 100,000 nodes (the scaling factor corresponding to an operation in $O(n^3)$ is then applied when varying the number of nodes), and consisting for 80% of a Library phase, and 20% of a General phase. We set the duration of the complete checkpoint and rollback ($C$ and $R$, respectively) to 1 minute when 100,000 nodes are involved, and we scale this value linearly with the total amount of memory, when varying the number of nodes. The MTBF at 100,000 nodes is set to 1 failure every day, and this also scales linearly with the number of components. The ABFT overheads, and the downtime, are set to the same values as in the previous section, and 80% of the application memory ($M_L$) is touched by the Library phase.

Given these parameters, Figure 9 shows (i) the relative waste of PurePeriodicCkpt, BiPeriodicCkpt, and ABFT&PeriodicCkpt, as a function of the number of nodes, and (ii) the average number of faults that each execution will have to deal with to complete. The expected number of faults is the ratio of the application duration by the platform MTBF (which decreases when the number of nodes increases, generating more failures). The fault-free execution time increases with the number of nodes (as noted above), and the fault-tolerant execution time is also increased by the waste due to the protocol. Thus, the total execution time of PurePeriodicCkpt or BiPeriodicCkpt is larger at 1 million nodes than the total execution time of ABFT&PeriodicCkpt at the same scale, which explains why more failures happen for these protocols.

When comparing BiPeriodicCkpt and PurePeriodicCkpt, one can see the benefit of application-level checkpointing, which spares about 20% of the checkpoint time during 80% of the checkpoints: this benefit shows up by a small linear reduction of the waste for BiPeriodicCkpt. However, both approaches perform similarly with respect to the number of nodes in this weak-scaling experiment.

Up to approximately 100,000 nodes, the fault-free overhead of ABFT negatively impacts the waste of the ABFT&PeriodicCkpt approach, compared to BiPeriodicCkpt or PurePeriodicCkpt. Because the MTBF on the platform is very large compared to the application execution time (and hence to the duration of each Library phase), periodic checkpointing approaches have a very large checkpointing interval, introducing very few checkpoints, thus a small failure-free overhead. Because failures are rare, the cost due to time lost at rollbacks does not overcome the benefits of a small failure-free overhead, while the ABFT technique must pay the linear overhead of maintaining the

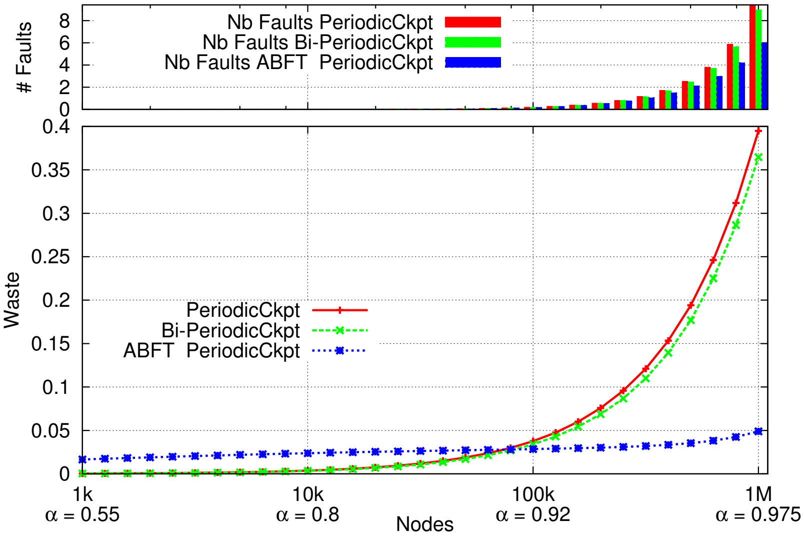


Figure 10: Total waste for ABFT&PeriodicCkpt, BiPeriodicCkpt and PurePeriodicCkpt, when considering the weak scaling of an application with variable ratio of time spent in a Library routine.

redundancy information during the whole computation of the Library phase.

Once the number of nodes reaches 100,000, however, two things happen: failures become more frequent, and the time lost due to failures starts to impact rollback recovery approaches. Thus, the optimal checkpointing interval of periodic checkpointing becomes smaller, introducing more checkpointing overheads. During 80% of the execution, however, the ABFT&PeriodicCkpt approach can avoid these overheads, and when they reach the level of linear overheads due to the ABFT technique, ABFT&PeriodicCkpt starts to scale better than both periodic checkpointing approaches.

All protocols have to resort to checkpointing during the General phase of the application. Thus, if failures hit during this phase (which happens 20% of the time in this example), they will all have to resort to rollbacking and lose some computation time. Hence, when the number of nodes increases and the MTBF decreases, eventually, the time spent in rollbacking and re-computing, which is linear in the number of faults, will increase the waste of all algorithms. However, one can see that this part is better controlled by the ABFT&PeriodicCkpt algorithm.

Next, we consider the case of an unbalanced General phase: consider an application where the Library phase has a cost $O(n^3)$ (where $n$ is the problem size), as above, but where the General phase consists of $O(n^2)$ operations. This kind of behavior is reflected in many applications where matrix data is updated or modified between consecutive calls to computation kernels. Then, the time spent in the Library phase will increase faster with the number of nodes than the time spent in the General phase, varying $\alpha$. This is what is represented in Figure 10. We took the same scenario as above for Figure 9, but $\alpha$ is a function of the number of nodes chosen such that at 100,000 nodes, $\alpha = T_L^{\text{final}}/T^{\text{final}} = 0.8$, and everywhere, $T_L^{\text{final}} = O(n^3) = O(\sqrt{x})$, and $T_{PC}^{\text{final}} = O(n^2) = O(1)$. We give the value of $\alpha$ under the number of nodes, to show how the fraction of time spent in Library phases increases with the number of nodes.

The PurePeriodicCkpt protocol is not impacted by this change, and behaves exactly as in Figure 9. Note, however, that $T^{\text{final}} = T_L^{\text{final}} + T_{PC}^{\text{final}}$ progresses at a lower rate in this scenario than in the previous scenario, because $T_{PC}^{\text{final}}$ does not increase with the number of nodes. Thus, the average number of faults observed for all protocols is much smaller in this scenario. Because more and more time (relative to the duration of the application) is spent in the Library phase, where 20% of the memory does not need to be saved, the BiPeriodicCkpt algorithm increases its

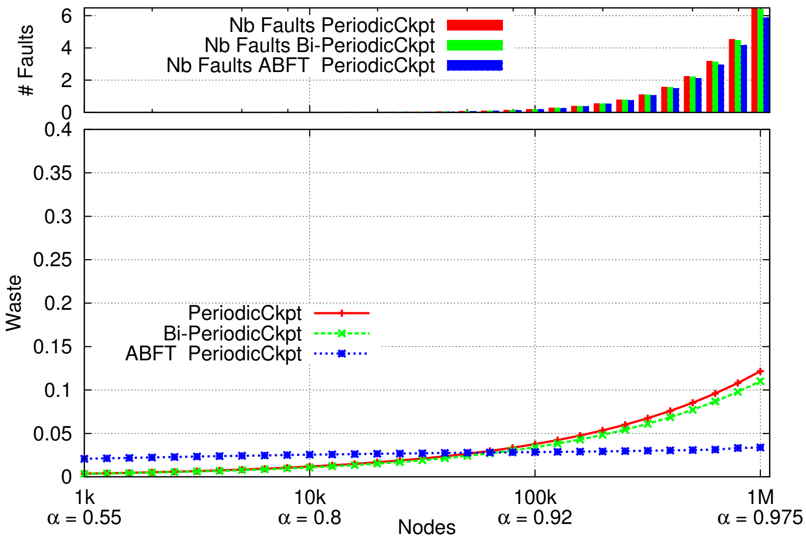


Figure 11: Total waste for ABFT&PeriodicCkpt, BiPeriodicCkpt and PurePeriodicCkpt, when considering the weak scaling of an application with variable ratio of time spent in a Library routine, and constant checkpointing time

benefit, compared to PurePeriodicCkpt: less overhead is paid for checkpoints that happen during Library phases, and the optimal period of checkpointing during these phases is longer. The cost of failures, however, remains the same, since the state of the entire application (Library memory, and Remainder memory) must be restored at rollback time.

The efficiency on ABFT&PeriodicCkpt, however, is more significant. The latter protocol benefits from the increased $\alpha$ ratio in both cases: since more time is spent in the Library phase, periodic checkpointing is de-activated for relatively longer periods. Moreover, this increases the probability that a failure will happen during the Library phase, where the recovery cost is greatly reduced using ABFT techniques. Thus, ABFT&PeriodicCkpt is capable of mitigating failures at a much smaller overhead than simple periodic checkpointing, and more importantly with better scalability.

In both previous evaluations, we have always considered the checkpointing (and rollback recovery) time proportional to the global amount of memory that needs to be saved in these checkpoints. This is realistic, if the checkpoint needs to be stored in a remote place, to guarantee its availability after a failure occurs. In this case, the interconnect (or the bandwidth capacity of the disks) eventually becomes a bottleneck, and the saving time becomes proportional to the number of computing resources saving their state simultaneously. To mitigate the negative effect of this bottleneck, system designers are studying a couple of alternative approaches. One consists of featuring each computing node with local storage capability, ensuring through the hardware that this storage will remain available during a failure of the node. Another approach consists of using the memory of the other processors to store the checkpoint, pairing nodes as “buddies,” thus taking advantage of the high bandwidth capability of the high speed network to design a scalable checkpoint storage mechanism [31, 32, 33, 34].

Thus, one might think reasonable to speculate that the checkpoint storage time will not increase with the number of nodes, but will actually remain constant. This is the scenario we contemplate in Figure 11. The scenario is identical to the previous scenario of Figure 10, but the checkpoint time ($C$) and rollback recovery time ($R$) are both independent of the number of nodes that checkpoint, and is fixed at 60s. One can see a noteworthy benefit on both periodic checkpointing protocols: even at 1 million nodes, the waste due to the protocols and the few faults that do occur during the execution (up to 6 failures on average during the whole execution) both add up to below 15%. At

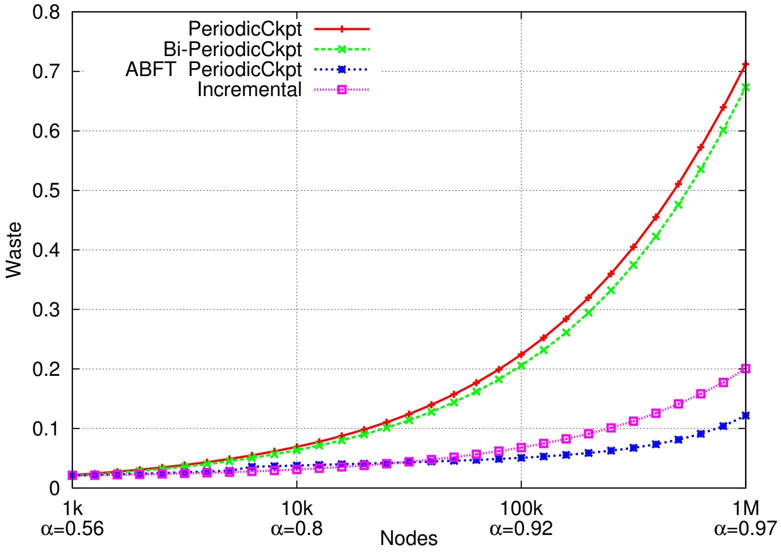


Figure 12: Total waste for ABFT&PeriodicCkpt, Inc&PeriodicCkpt, BiPeriodicCkpt and PurePeriodicCkpt, when considering the weak scaling of an application with variable ratio of time spent in a Library QR routine.

the same time, the ABFT technique continues to introduce its constant overhead (due to additional computation) during the whole execution, and appears to present a waste that is almost constant when the number of nodes increases.

Figure 11 shows that PurePeriodicCkpt and BiPeriodicCkpt are less efficient than ABFT-&PeriodicCkpt at 1 million nodes, despite the perfectly scalable checkpointing hypothesis. To reach comparable performance, we must reduce checkpointing overhead by a factor of 10 and use $C = R = 6s$. Such low figures can only be achieved through new hardware (like NVRAM), and new hierarchical checkpointing protocols.

### 6.4 Incremental checkpointing

In our comparison between ABFT-hybrid and checkpointing, the last optimization that we consider is to employ incremental checkpointing during the Library phases (as detailed in Section 5). With incremental checkpointing, the checkpoint data volume depends on the memory write access pattern of the application, thus the computation of the optimal repartition of incremental checkpoints requires knowledge of the algorithmic features of the Library phases. We present the results for the right looking QR factorization (as described in Section 5.1).

In Figure 12, we consider an instantiation of an application that alternates General phases and QR factorizations; the repartition of General versus QR phase durations $\alpha$ is set to 0.8 at 10,000 nodes, and varies according to an imbalance ratio $n^3/n^2$ (similarly to the setup employed in Figure 10). The application dataset is scaled to the number of nodes to remain constant at 16GB per node. We consider the case of a machine with one I/O node per 100 compute nodes, and an individual I/O node bandwidth of 80Gb/s; the resultant values for C and R are approximately 25s at any scale. The failure rate is set to 1 failure per day at 10,000 nodes (and scaled linearly with the number of nodes).

As can be observed, the more conservative hypothesis made in this deployment regarding the storage capacity and MTBF of components, result in a sharp increase in the waste incurred by periodic checkpointing approaches. The incremental checkpointing approach departs from this behavior and exhibits a better waste profile even for millions of nodes: the waste is reduced to less than 20%. Yet the ABFT composite approach still outperforms it by a significant margin. One can note

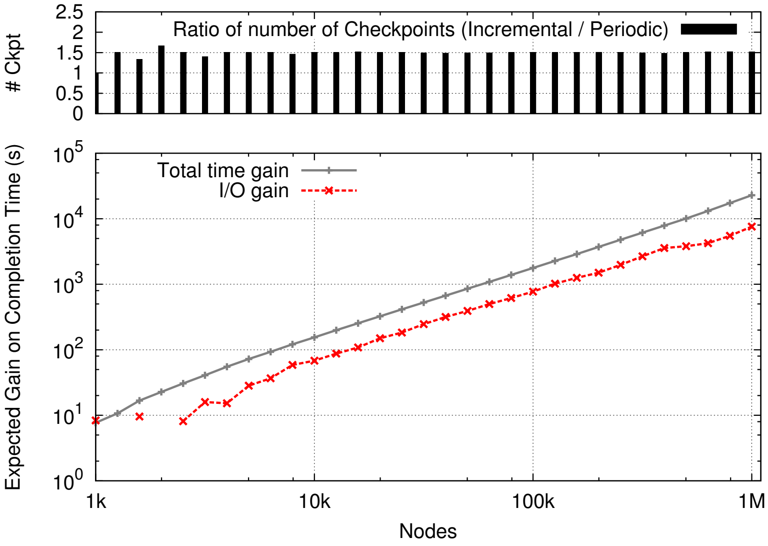


Figure 13: Benefits from employing Inc&PeriodicCkpt over BiPeriodicCkpt in terms of completion time and I/O volume (QR routines).

that leveraging on knowledge about the behavior of the computational routines permits outstanding gains in both cases, in the case of incremental checkpointing by better scheduling the checkpoints, and in the case of ABFT by reducing the I/O pressure.

In order to further quantify the gain achieved by the dynamic programming scheduling of incremental checkpoints, we compare the checkpoint costs incurred during the Library phase versus the costs generated by a periodic approach. Note that since we compare only the costs during the Library phase in this figure, we take the BiPeriodicCkpt approach, and ignore the Remainder memory altogether. Figure 13 presents this comparison (for the same deployment parameters presented above). The top bar-graph outlines the difference in the number of checkpoints taken between Inc&PeriodicCkpt and BiPeriodicCkpt. Because in Inc&PeriodicCkpt, checkpoints are smaller and thus less costly, the dynamic programming schedules more checkpoints, thereby improving the resiliency of the application by reducing the average rollback incurred by failures. Noteworthy, the distribution of these checkpoints over time are not periodic: as the operation progresses, it goes faster to achieve each task and a lower amount of memory is updated. The dynamic program finds the optimal position of the checkpoints to become more and more frequent, reducing the risk of re-execution in case of failure.

The second graph of Figure 13 presents the time difference and I/O cost difference between BiPeriodicCkpt and Inc&PeriodicCkpt. Although the incremental approach takes more checkpoints, the overall checkpointing volume is actually reduced, which translates into a major reduction in the overall runtime. Overall, and as is the case for ABFT&PeriodicCkpt, leveraging knowledge about the computational routines behavior is highly beneficial. Thanks to knowing the access pattern and thereby the volume of each checkpoints taken at a particular step, the dynamic programming of incremental checkpoints can reduce drastically the I/O pressure, the average rollback incurred by each failures, which translates into massive gains on the overall runtime.

# 7 Conclusion

In this paper, we have formalized and quantified a novel method of composing fault tolerance approaches for applications that alternate between routines for which advanced, algorithm-aware fault tolerance methods can be deployed, and opaque sections for which no particular features are prominent to leverage from a fault tolerance approach. In the resultant composition, each of these sections is protected by its own mechanism, ABFT or incremental checkpointing in one case and generic checkpoint/restart in the other. A performance model has been derived for such methods and thoughtfully validated using a simulator developed for this scope. We have compared our composite approach with a traditional periodic checkpointing approach using rollback and recovery, under different plausible scenarios. Our model predicts that the cost of a "checkpoint only" approach will maintain a reasonable overhead only under highly optimistic assumptions, where the checkpointing cost stagnates when the number of computational resources increases. Under more realistic assumptions, where the checkpointing cost increases with the number of resources, the composite approach will provide significantly greater benefits compared with checkpoint/restart, by minimizing the waste and thus increasing the platform throughput. We also consider an optimization of checkpoint/restart which employs incremental checkpointing and leverage an intimate knowledge of the computing routine memory access pattern to devise an optimal dynamic programming schedule of the checkpoints. Despite the great improvements demonstrated over basic periodic checkpoint/restart, our weak scalability study shows that the gain of the ABFT composite approach will continue to grow with the increase in the number of computing resources, making it a more plausible and desirable approach at very large scale.

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